Liquidity-Adjusted Value-at-Risk for Alternative Assets

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Investment in alternative assets has become popular among pension funds in the recent years, due to high rewards in the long run and low correlation with traditional assets.¹ As of July 2009, alternative assets have taken up 17 percent of all pension fund assets in the world, compared with seven percent ten years ago.²

Even during the financial crisis, investment in alternative assets did not decrease significantly, in contrast to equity. Based on a recent study by Towers Watson together with Financial Times, “Alternative assets managed on behalf of pension funds globally by the Top 100 managers fell by around 1 percent during 2008 compared to 40 percent growth the previous year.”³ There may be several reasons to explain this phenomenon. First, given the poor performance of equity during the crisis, there was a demand from fund managers for diversification. Moreover, alternative assets are characterized by illiquidity, so the investment weight in alternative assets did not fluctuate a lot. Meanwhile, due to the illiquidity restrictions alternative assets usually have higher liquidity risk premium. Hence, they may be performing better than equities during crisis.

The so-called illiquidity restrictions usually refer to the redemption restrictions, such as “lock-up” and “redemption gate” for private equity and hedge funds, whose objective is to provide the fund manager enough time to adjust the investment strategy without affecting the fund value significantly. The lock-up is the minimum holding period, during which the investor cannot withdraw his money. The duration of lock-up may vary from one month to two or three years, which may depend on the specified investment strategy of the fund. Redemption gate is usually employed by hedge funds and private equity to “limit the percentage of fund capital that can be withdrawn on the fund’s scheduled redemption date.”⁴ For an investor, lock-up and redemption may result in liquidity risk, that is, extra risk exposure due to constrained holding.

Some people⁵ argue that liquidity risk may not be a big problem for long term investment like pensions, even when they have regular liability to meet, and benefits from unreasonably high liquidity risk premium can well set off any negative impact from the lock-up effect. But a scrutiny will reveal that the liquidity problem exists even in the long run. Firstly, difficulty in getting timely and correct valuation will affect the efficiency of rebalancing.⁶ Secondly, balance sheet pressure during a crisis from use of a lower discount rate will make the present...
value of liabilities much higher, implying higher contribution from the sponsor for funding requirements.

In this article, we discuss the implication of illiquidity to the risk exposure of some alternative assets. We employ Value-at-Risk (VaR) as our risk measure for alternative assets, which measures the potential loss over a specified period under a worse situation. The general idea is as follows: if we measure the risk exposure of an alternative asset over a specified period, which is shorter than the required holding period that an investor has to take due to “lock-up” or “redemption gate”, the investor has to suffer the liquidity risk exposure for capital lock. Even though the investor realizes the potential loss in the following period will increase significantly, he will not be in the position to avoid the loss. As a result, the potential loss is larger than the loss over the specified period because of extra risk exposure. So when we measure the risk exposure over this specified period, it will be more reasonable to investigate the potential loss over the minimum holding period and discount the potential loss to the end time of the specified period (see figure 1).

**Figure 1: Liquidity adjusted Value-at-Risk**

Assume the initial value of alternative asset is $A_0$ and the return of alternative asset $R$ follows log normal distribution with mean $\mu$ and standard deviation $\sigma$. $T$ is the additional constrained holding period. $r$ is the risk-free rate. Then the liquidity-adjusted VaR ($L-VaR$) over $n$ period at confidence level $\alpha$ can be calculated by discounting the traditional Value-at-Risk (VaR) over $n+T$ periods from $t_2$ to $t_1$ as according to the following equation:

$$L-VaR_t = e^{-rt} \text{VaR}(A_0 e^{\sigma^2T})$$

We consider two cases for the constrained holding period $T$: (1) $T$ is a known constant; (2) $T$ is an increasing function of asset size. The two cases correspond to the lock-up and redemption gate of alternative asset respectively. Let us explain with two illustrative examples.

Suppose the initial capital is 100 million and there are two types of alternative assets: private equity and hedge funds. The expected yearly return of private equity is 10.5 percent with a
standard deviation as 42.4 percent. And the expected yearly return of the hedge fund is 3.8 percent with a standard deviation as 9.0 percent. We aim to investigate the VaR over one month with a 95 percent confidence level.

When we consider a lock-up, the constrained holding period T can be taken as a known constant. We compare the liquidity-adjusted VaR across different lock-up periods: 2 months, 3 months, 6 months and 1 year.

Table 1: Liquidity adjusted Value-at-Risk by different lock-up period

<table>
<thead>
<tr>
<th>Lock-up</th>
<th>PE VaR (million)</th>
<th>Hedge Fund VaR (million)</th>
<th>L-VaR (million)</th>
<th>Underestimate percent (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Month</td>
<td>17.85</td>
<td>3.93</td>
<td>24.75</td>
<td>5.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>39</td>
<td>36</td>
</tr>
<tr>
<td>3 Month</td>
<td></td>
<td></td>
<td>29.05</td>
<td>6.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>63</td>
<td>62</td>
</tr>
<tr>
<td>6 Month</td>
<td></td>
<td></td>
<td>38.20</td>
<td>11.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>114</td>
<td>191</td>
</tr>
<tr>
<td>12 Month</td>
<td></td>
<td></td>
<td>49.38</td>
<td>17.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>177</td>
<td>343</td>
</tr>
</tbody>
</table>

Source: Towers Watson calculations

Table 1 shows the results of liquidity adjusted VaR and ordinary VaR. It can be seen that the liquidity-adjusted risk exposure is positively correlated with the length of the lock-up period and the differences between VaR and L-VaR are very significant. The largest difference is more than 300 percent.

When we consider the redemption gate, the constrained holding period T should be treated as an increasing function of asset value. Suppose the total size of private equity and hedge fund are 1000 million. Investors' withdrawals are limited to a cap of 10 percent of total assets in every redemption date. And the redemption date is the last day of every month. The part above cap will be deferred to next redemption dates. Here we compare the liquidity-adjusted VaR across different asset sizes: 150 million, 250 million and 500 million. Correspondingly, the redemption will be deferred to two months, three months and five months.

As shown in table 2, VaR will underestimate the risk exposure more for a larger asset size. VaR as a percentage of initial asset is 18 percent for private equity, but the liquidity adjusted VaR will take up 20 percent, 23 percent, 28 percent of initial assets respectively, when initial asset increases from 150 million to 500 million. And the corresponding underestimate percent
increases from 13 percent to 57 percent. If the redemption date is of a lower frequency, such as quarterly or half yearly, the liquidity adjusted VaR will be larger and the underestimate degree of VaR will be more significant.

**Table 2: Liquidity adjusted Value-at-Risk by different initial asset**

<table>
<thead>
<tr>
<th>Initial Asset (million)</th>
<th>PE</th>
<th>Hedge Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value (million)</td>
<td>Percentage of initial asset (%)</td>
</tr>
<tr>
<td>VaR 150</td>
<td>26.77</td>
<td>18</td>
</tr>
<tr>
<td>L-VaR 150</td>
<td>30.24</td>
<td>20</td>
</tr>
<tr>
<td>Underestimate percent (%)</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>VaR 250</td>
<td>44.62</td>
<td>18</td>
</tr>
<tr>
<td>L-VaR 250</td>
<td>57.19</td>
<td>23</td>
</tr>
<tr>
<td>Underestimate percent (%)</td>
<td>28</td>
<td>27</td>
</tr>
<tr>
<td>VaR 500</td>
<td>89.24</td>
<td>18</td>
</tr>
<tr>
<td>L-VaR 500</td>
<td>140.37</td>
<td>28</td>
</tr>
<tr>
<td>Underestimate percent (%)</td>
<td>57</td>
<td>77</td>
</tr>
</tbody>
</table>

Source: Towers Watson calculations

Based on our analysis of the implication of lock-up and redemption gate of private equity and hedge fund, it is found that degree by which VaR is underestimated as compared to liquidity-adjusted VaR, will be quite significant and increase along with the length of lock-up and initial value of the asset, and decrease with lower frequency of the redemption date. It may be concluded that illiquidity may be an important issue to measure risk exposure. Bias of risk estimation will harm the efficiency of rebalancing the portfolio and brings balance sheet pressure for pension fund. One possible solution is to employ liquidity-adjusted VaR, which measures the potential loss over the minimum holding period and discounts the potential loss to the end time of our target period.


8 See footnote 4.